

Previously:

Set up a business teaching Algebra and Arithmetic skills to students using a combination of online Instruction and in person tutoring. We have determined that our profit for each algebra student will be \$2 and \$1.25 for each Arithmetic student.

Another piece of the problem is constraints. Students consume resources and we have to support our students. Algebra students are allotted 2 hours of computer instruction and 1 hour of tutoring each week. Arithmetic students have 1 hour of instruction and 1.5 hours of tutoring allowed to them. Our company has the resources to handle a total of 300 hours of computer instruction and 200 hours of in person tutoring each week. This sets limits not only on the total number of students we can support, but also the mix of students we can support.

Objective Function: Maximize $P=2x + 1.25y$

Variables: $x = \#$ of Algebra students that sign up
 $y = \#$ Arithmetic students that sign up

Constraints: $2x + 1y \leq 300$ (computer Instruction hours)
 $1x + 1.5y \leq 200$ (tutoring hours)

Excel has a built in function called **Solver** that can be used to solve linear programs. (It is an Add-In, so if you do not find it under Data you may need to load it.) The way it works is that you provide the equation to be optimized, an initial guess for the solutions, and all of the constraint equations, all as interactive cells. Then Solver runs an algorithm and comes up with an answer (if one exists). As always, you need to interpret what it tells you!

Presentation is important, so use a format similar to this:

The screenshot shows the Microsoft Excel Solver and Sensitivity Demo window. The Solver Parameters dialog box is open, and the Solver Options tab is selected. The Solver Parameters dialog box shows the following settings:

- Set Objective:** \$D\$8 (Profit: $2x + 1.25y$)
- To:** Max Of
- By Changing Variable Cells:** \$D\$7:\$D\$8 (Algebra students: x , Arithmetic students: y)
- Subject to the Constraints:**
 - \$E\$11:\$E\$12 <= \$F\$11:\$F\$12 (Computer hrs: $2x + 1y \leq 300$, tutor hrs: $1x + 1.5y \leq 200$)
- Make Variable Cells Non-Negative:** Checked
- Solving Method:** GRG Nonlinear Engine
- Help:** Solver Help

The Solver Parameters dialog box is shown below:

Cell Reference	Constraint	Comparison Operator	Constant Value
\$E\$11:\$E\$12	$2x + 1y \leq 300$	<=	300
\$E\$12:\$E\$12	$1x + 1.5y \leq 200$	<=	200

The Solver Options dialog box is also visible, showing the following options:

- Make Unconstrained Variables Non-Negative:** Checked
- GRG Nonlinear Engine:** Selected
- Load/Save:** Solver Load/Save options
- Help:** Solver Help

The objective function and the constraints are all functions of the variable cells. Put in any values for the variables to start. 1 or 0 are good choices.

When it is set up, go to the Data tab, and then click on the Solver tool.

Need to designate the Objective function, whether to Maximize or Minimize, which cells are the variables, and then use Add to enter the constraints.

The image shows the Solver Parameters dialog box with several red callout boxes providing instructions:

- Set Objective:** A callout box points to the cell address `D4` with the text: "The cell address that has our profit equation".
- To:** A callout box points to the **Max** radio button with the text: "Click on the Max bullet as we want to maximize profit".
- By Changing Variable Cells:** A callout box points to the cell address `D7:D8` with the text: "The cell addresses of our variables".
- Subject to the Constraints:** A callout box points to the list of constraints (`D11 <= F11` and `D12 <= F12`) with the text: "Use the Add button to enter the addresses of the constraint equations and values.".
- Make Unconstrained Variables Non-Negative:** A callout box points to the checked checkbox with the text: "Checking this box saves the trouble of entering the $x, y \geq 0$ constraints".
- Select a Solving Method:** A callout box points to the dropdown menu set to "Simplex LP" with the text: "VERY IMPORTANT! Use the drop down menu to select Simplex LP".

At the bottom of the dialog, there are buttons for **Help**, **Solve**, and **Close**.

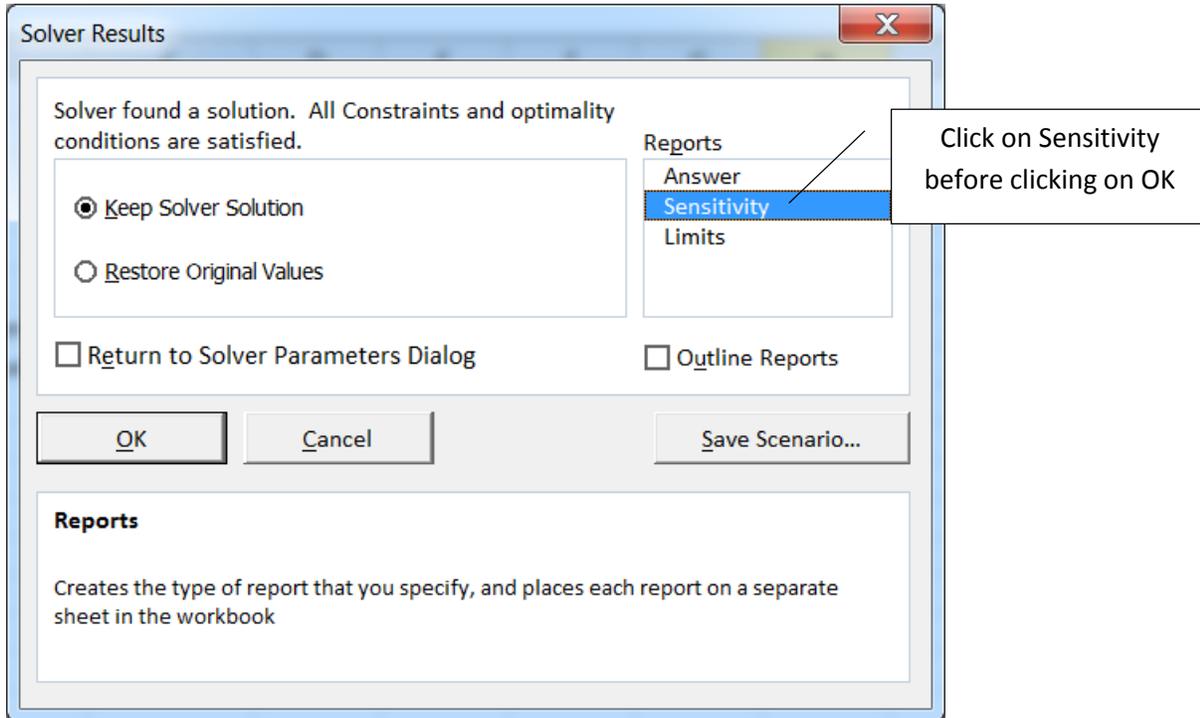
Steps to use Solver:

1. Select the Data tab in the toolbar.
2. Choose Solver. (If it isn't there, you need to choose Add-Ins and check the Solver Add-In.)
3. Answer the questions in the dialog box:
 - Target Cell:(the cell containing the objective function: here it's profit amount
 - Equal to: (check appropriate option, max. or min.)
 - By changing: (cells containing your values for the variables)
 - Subject to: (the cells that contain the formulas and values of the constraints. Use the Add button to enter them)
4. Check the box "Make unconstrained variables non-negative" and for the drop down menu for "Select a Solving Method" select Simplex LP
5. Click the Solve button.

Be sure to watch the videos for this section!

Sensitivity Reports

Would tweaking the profit model alter the optimal solution? If we have access to a little more of a resource is an increase in Profit worth the extra expense? To answer these questions, run solver again, but this time select the Sensitivity Report:



This will insert a new worksheet in your work book, Labeled Sensitivity Report 1:

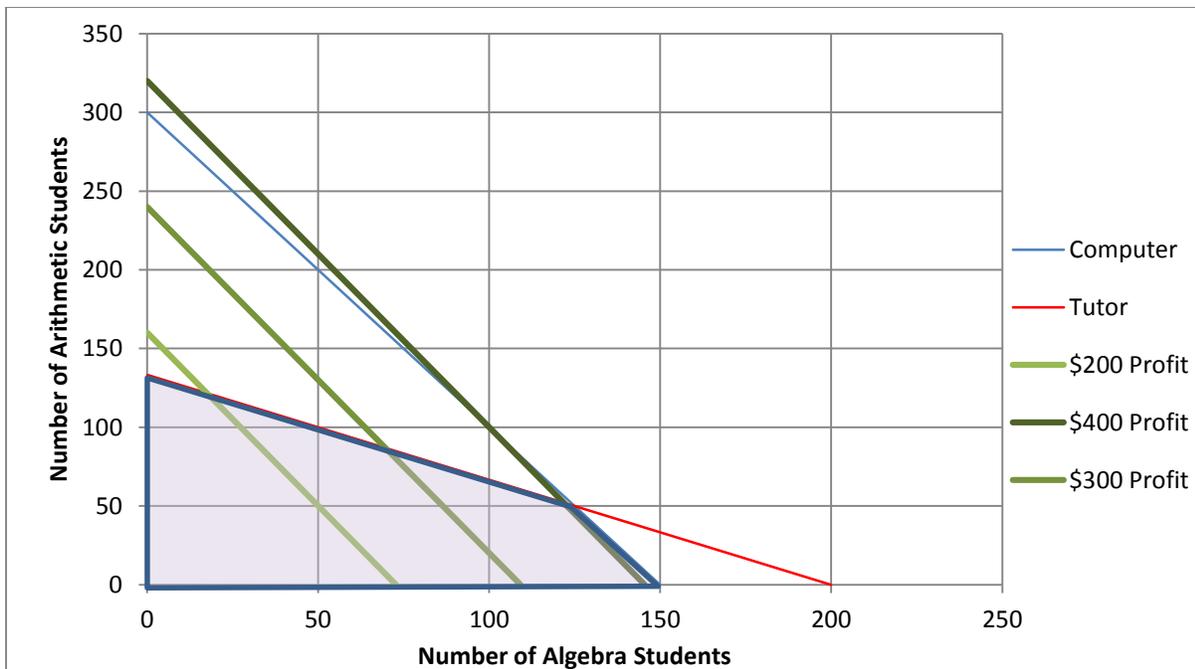
Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$D\$7	x = # Algebra students	125	0	2	0.5	1.166666667
\$D\$8	y = # Arithmetic students	50	0	1.25	1.75	0.25

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$D\$11	2x + 1y <= 300	300	0.875	300	100	166.6666667
\$D\$12	1x + 1.5y <= 200	200	0.25	200	250	50

Top section: $x = \#$ of alg. students. The Objective coefficient is 2, which is our profit for each algebra student. The allowable increase is 0.5 and the Allowable Decrease is 1.167. That means (assuming the coefficient of y is not changed) the coefficient of x could be decreased by 1.167 to as low as .833 or increased by as much as .5 to as high as 2.5 and the solution of (125, 50) would not change.

With a profit function of $2x + 1.25y = P$, our best “product mix” is to have a mix of students: 125 algebra and 50 arithmetic students. If we were to alter our business model so that the profit per algebra student dropped lower than .833 or higher than 2.5 then our profit isolines would tilt enough so that our best solution would jump to another corner of the feasible region.

See the graph below for profit isolines of $2.75x + 1.25y = P$. It appears that 150 algebra students and no arithmetic students would maximize our profit now. Which may or may not be a good thing for our business, but either way it would be a significant change.



The next line on the sensitivity report is: $y = \#$ of arithmetic students. The Objective coefficient is 1.25, the profit on arithmetic students and the allowable increase is 1.75, allowable decrease is 0.25. So the coefficient on y (profit on an arithmetic student) could be as low as 1.00 and as high as 3.00 without changing our mix of 125 algebra students and 50 arithmetic students as being the optimal solution.

	A	B	C	D	E	F	G	H	I	J
1	Microsoft Excel 14.0 Sensitivity Report									
2	Worksheet: [Solver and Sensitivity Demo.xlsx]Original set up									
3	Report Created: 8/18/2014 5:07:24 PM									
4										
5										
6	Variable Cells									
7										
8				Final	Reduced	Objective	Allowable	Allowable		
9	Cell	Name	Value	Cost	Coefficient	Increase	Decrease			
10	\$D\$7	x = # Algebra students	125	0	2	0.5	1.166666667			
11	\$D\$8	y = # Arithmetic students	50	0	1.25	1.75	0.25			
12	Constraints									
13			Final	Shadow	Constraint	Allowable	Allowable			
14	Cell	Name	Value	Price	R.H. Side	Increase	Decrease			
15	\$D\$11	2x + 1y <=300	300	0.875	300	100	166.6666667			
16	\$D\$12	1x + 1.5y <=200	200	0.25	200	250	50			
17										
18										

In the lower section of the sensitivity report look at the line where $2x + 1y \leq 300$, that is the constraint for computer instruction. Read across to the Shadow Price for that constraint, which is 0.875. That means that for each additional hour of computer instruction we can come up with, profit would rise by \$.875 this is limited by the allowable increase of 100 hours. Could we get an additional 100 hours of computer instruction for less than \$.875? If so, then it would be worth it.

The shadow price on the tutoring ($1x + 1.5y \leq 200$) is \$.25, an increase of one hour of tutoring would increase profit by \$.25. It is unlikely that we can get an additional hour of tutoring for that price.