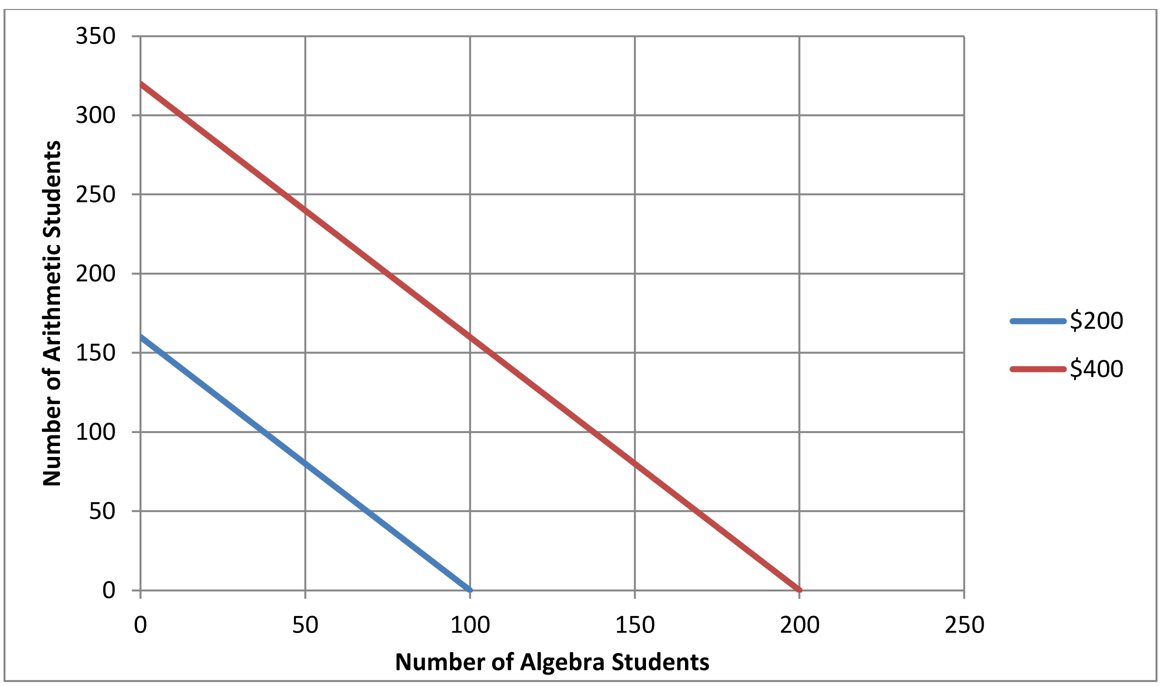
MTH245 Unit 2 Module 4 Linear programming and Optimizing Systems

We will take the skills you developed in the last module and see where we can apply them. The scenario we will look at is a business teaching Algebra and Arithmetic skills to students using a combination of online Instruction and in person tutoring. We have set up a business model such that our profit for each Algebra student will be \$2 and \$1.25 for each Arithmetic student. We are going to determine the optimal mix of students to register in order to maximize our profit but also stay within our business constraints.

Profit isolines show the different combinations of students that yield the same total profit for our company. If we let x = # of algebra student and y = # of arithmetic students then our profit would be $2x + 1.25y = P$

For example, a profit of \$200 could be had by having 100 algebra students and no arithmetic students. But we could have 160 arithmetic students and no algebra students and still have the same profit of \$200. (The blue line below, $2x + 1.25y = 200$)

For a \$400 Profit our intercepts would be 320 arithmetic & 0 algebra students, and 0 arithmetic students & 200 algebra students. (The red line below, $2x + 1.25y = 400$)



The more students we have the more profit we make, and we make more profit on the Algebra students. Isn't the answer just to sign up as many algebra students as possible?

No, because another piece of the problem is constraints. Students consume resources and we have to support our students. We will be using a combination of computer and in person tutoring to provide instruction, and we have some limitations on those resources.

Algebra students are allotted 2 hours of computer instruction and 1 hour of tutoring each week. Arithmetic students have 1 hour of instruction and 1.5 hours of tutoring allowed to them. Our company has the resources to handle a total of 300 hours of computer instruction and 200 hours of in person tutoring each week. This sets limits not only on the total number of students we can support, but also the mix of students we can support.

Possible Scenarios:

If we wanted to expend all of our computer resources on the algebra students, we could have 150 algebra students and no arithmetic students. In this scenario those students would also use 150 hours of tutoring, and there would be 50 hours of tutoring that went to waste.

# of algebra students	# of arithmetic students	Computer Use: 2 hrs per algebra & 1 hr per arith	Tutor Use: 1 hr per algebra & 1.5 hrs per arith.	Profit: \$2 per alg & \$1.25 per arith
150	0	300 hrs	150 hrs	\$300

If we wanted to have 100 algebra students and 100 arithmetic students, we have the computer time but our tutoring hours would be exceeded. (We have 200 hours of tutoring time available.) So this is not a feasible solution.

# of algebra students	# of arithmetic students	Computer Use: 2 hrs algebra & 1 hr arithmetic	Tutor Use: 1 hr algebra & 1.5 hrs arith.	Profit: \$2 per alg & \$1.25 per arith
150	0	300 hrs	150 hrs	\$300
100	100	300 hrs	~~250 hrs~~	

50 algebra and 100 arithmetic is feasible: that would use 2(50) + 1(100) = 200 hours of computer time, and 1(50) + 1.5(100) = 200 hours of tutoring. This combination would use all of the tutoring resources and have some excess computer time. But our profit is not as high as 150 algebra students and no arithmetic.

# of algebra students	# of arithmetic students	Computer Use: 2 hrs algebra & 1 hr arithmetic	Tutor Use: 1 hr algebra & 1.5 hrs arith.	Profit: \$2 per alg & \$1.25 per arith
150	0	300 hrs	150 hrs	\$300
100	100	300 hrs	~~250 hrs~~	
50	100	200 hrs	200 hrs	\$225

Guessing and testing will take forever; there must be a better way!

Let's focus on our constraints. We have at most 300 hours of computer instruction and 200 hours of tutoring available. Keeping our variables: x = # of algebra students, y = # of arithmetic students

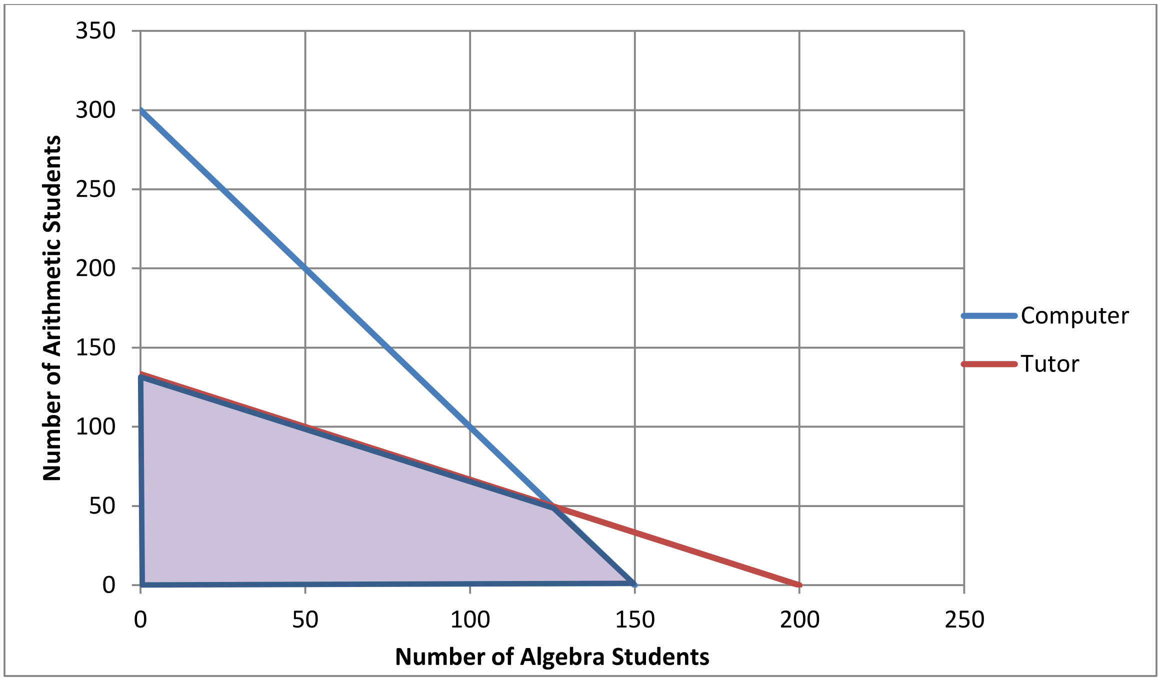
	Algebra	Arithmetic	Resources
Computer Instruction	2	1	300
In Person Tutoring	1	1.5	200

Each algebra student uses 2 hours of instruction and each arithmetic student has 1 hour of instruction. So our constraint for the instruction is that $2x + 1y <= 300$

Each algebra student uses 1 hour of tutoring and each arithmetic student uses 1.5 hours of tutoring. We cannot exceed 200 hours of tutoring. $1x + 1.5\,y <= 200$ models our constraint due to tutoring.

We have two additional constraints on our variables. Can we have a negative number of students in either category? No, so $x => 0, y => 0$

We can then graph the feasible region for our constraints and determine the corner points of the feasible region. This region has 4 corners: (0,0), (150,0), (0, 133) and (125, 50)



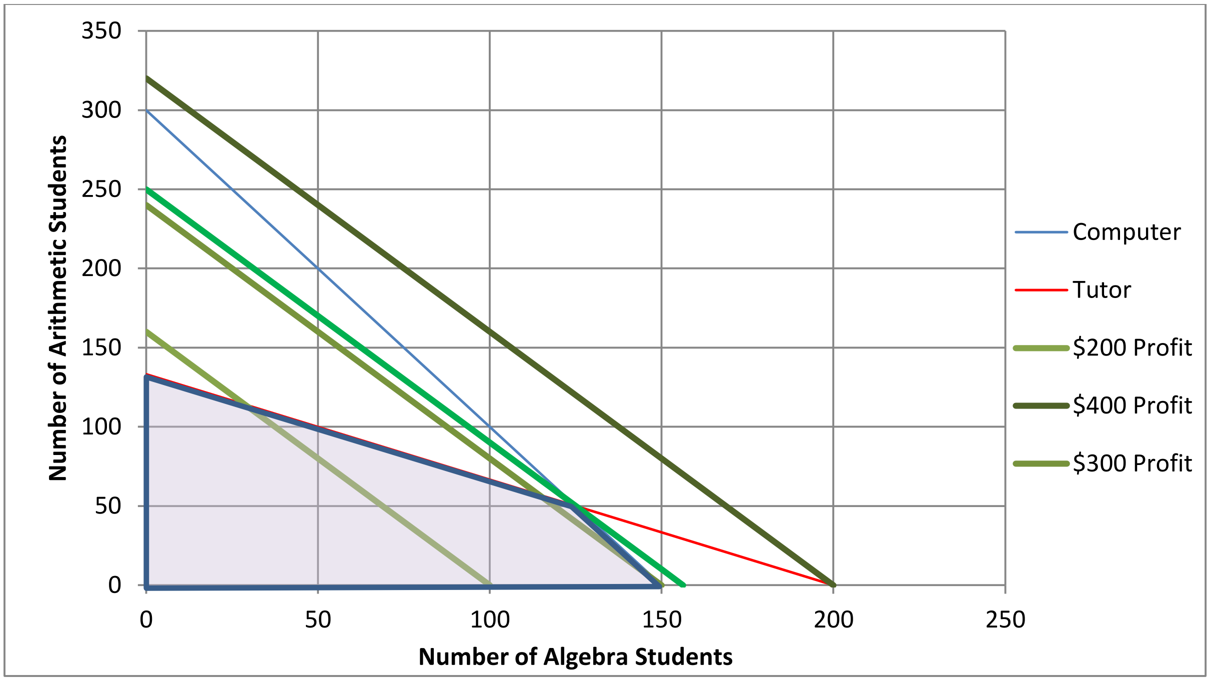
Why do we care about the corner points? If a solution to the system exists, it will occur at a corner point.

Now we will systematically check each corner point of our feasible region in the profit function:

X	Y	P=2x + 1.25y
0	0	0
150	0	$300
0	133	$166.25
125	50	$312.50

Our best mix of students is to have 125 algebra students and 50 arithmetic students.

Below is a graph for our profit equation $2x + 1.25y = P$ for various values of P, and it is overlaid with our feasible region that was determined from our constraints. Our goal in linear programming is to find a solution that both satisfies our constraints and maximizes our profit. From the graph you can see that the corner point (125, 50) will reach the profit isoline that is furthest out--the profit isoline where P would be 312.5.



The elements you will need to determine before you start solving optimization problems are:

- The equation to be optimized
- The variables
- The constraint equations

Important! The equation to be optimized and the constraint equations must be using the same set of variables. The values of variables are what you are trying to find, so think carefully about what your variables should be before writing any equations.

Another example: A student has work study job as a classroom aide that pays $10 an hour. In addition, he tutors privately for $15 an hour. The student can work no more than 20 hours a week, or he will not have enough time for his own classes. Also, he must work at least 3 hours a week as the classroom aide. How many hours a week should the student work at each job in order to maximize his earnings?

We are asked to determine how many hours the student should work at each job. Those will be our variables:

a = hours working as a classroom aide, t = hours working as a private tutor.

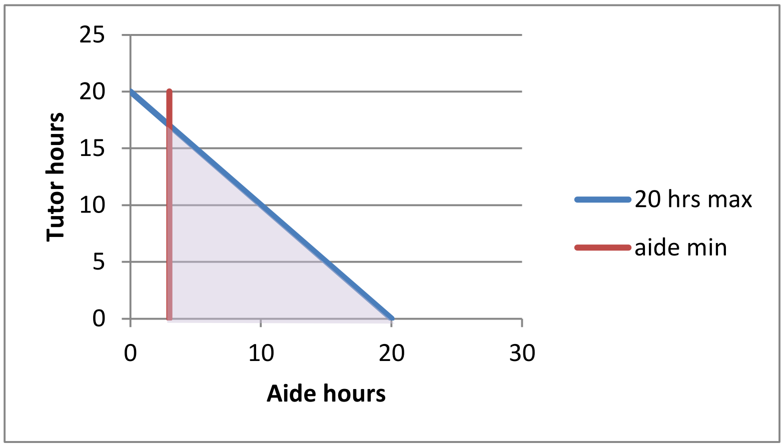
He wishes to maximize his earnings, and he is paid $10 as an aide and $15 as a tutor:

Maximize: Earnings = 10a + 15t

Constraints:

- He can work no more than 20 hours a week: a + t <= 20
- He must work at least 3 hours as an aide: a>=3
- He cannot work a negative number of hours: a, t >=0

To solve, we will first graph the constraints:



There are three corner points: (3, 0), (20, 0) and (3, 17)

Check each corner point in the equation we wish to optimize:

Aide	Tutor	Earn: 10a+15t
3	0	$30
20	0	$200
3	17	$285

The student will earn the most if he works the minimum 3 hours a week as an aide and the remaining 17 hours as a tutor.