

MTH245 Day 5 Piecewise Linear Functions

Some situations require a changing model. Look at a copy shop. If they charge \$.05 per copy then a model for the amount charged for n copies would be $C(n) = .05n$.

But most shops offer a discount on large orders; this shop charges \$.05 for orders up to 100, and then \$.03 for orders beyond that. One way to think about this would be:

If the number of copies ordered is ≤ 100	If the number of copies ordered is > 100 , the first 100 copies costs us \$5, then we are charged 0.03 for all copies beyond 100.
Model $C(n) = 0.05n$, for $n \leq 100$	Model $C(n) = 5 + 0.03(n - 100)$, for $n > 100$

There is a way to combine these two models into one model (it will look a little rinky-dink)

$$C(n) = \begin{cases} 0.05n, & \text{if } n \leq 100 \\ 5 + 0.03(n - 100), & \text{if } n > 100 \end{cases} \quad \text{This called a piecewise (linear) function}$$

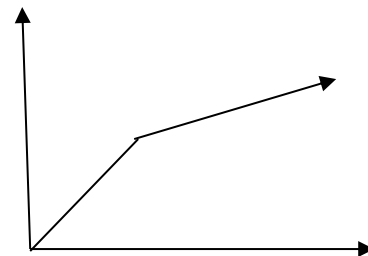
Graphically, we have a function formed by pieces of the two formulas in our function. The domain of each piece is given by the "if" statement and that tells us when to use which part.

note: Don't bother simplifying the $100 + 0.03(n - 100)$, Excel has no problem with it and it makes more sense in terms of the problem to express it like that.

Let's talk about "if". "If" is an operation in logic, and in Excel. An "IF" statement might be along the lines of "If I do my homework then I will pass the class, otherwise I will fail."

Format is: IF(something is true, then something happens, otherwise something else will happen)

Simple example: $=IF(x \leq 5, y = 0, y = 10)$ $f(x) = \begin{cases} 0, & \text{if } x \leq 5 \\ 10, & \text{if } x > 5 \end{cases}$ *Do by hand, then in Excel*



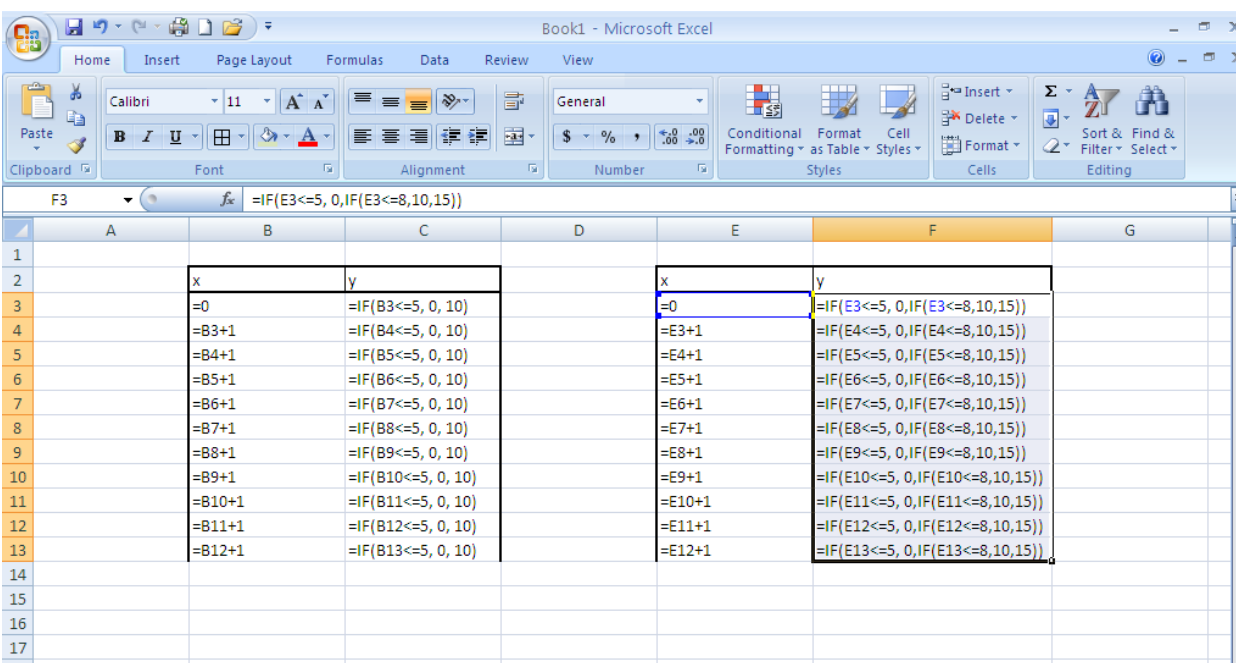
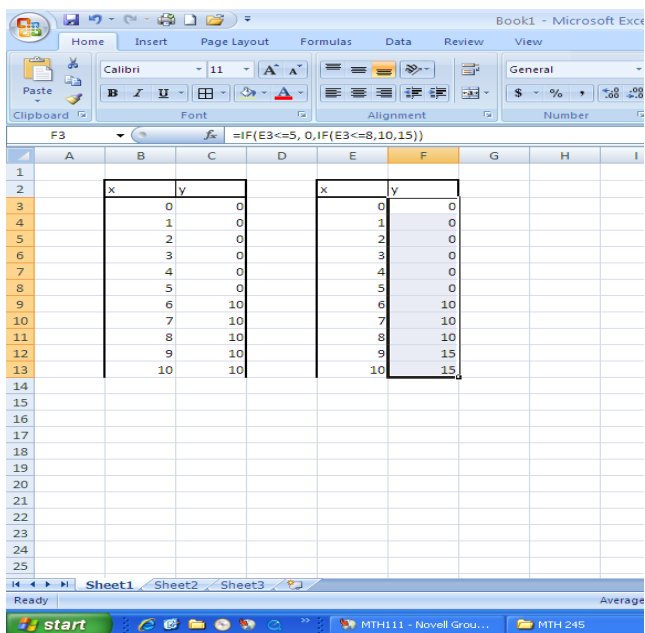
What about something more complicated?

$$f(x) = \begin{cases} 0, & \text{if } x \leq 5 \\ 10, & \text{if } 5 < x \leq 8 \\ 15, & \text{if } x > 8 \end{cases} \quad \text{We need to nest our IF statement. Starts off the same:}$$

$=IF(x \leq 5, y = 0, \text{then what?})$ $=IF(x \leq 5, y = 0, IF(x \leq 8, y = 10, \text{or else what?}))$

$=IF(x \leq 5, y = 0, IF(x \leq 8, y = 10, 15))$ *Do by hand, then in Excel*

Note, why don't we have to use $IF(x \leq 5, y = 0, IF(5 < x \leq 8, y = 10, 15))$?????



Back to the copy shop $C(n) = \begin{cases} 0.05n, & \text{if } n \leq 100 \\ 5 + 0.03(n - 100), & \text{if } n > 100 \end{cases}$ Translate into an IF statement

IF $n \leq 100$, then use $c = .05n$, otherwise use $c = 5 + 0.03(n-100)$

$=IF(n \leq 100, .05n, 5 + 0.03(n-100))$ very succinct, but note that the order of the arguments is critical.

One more, we get an additional discount if the order is more than 250 copies, those copies will only be \$0.02. First write out piecewise formula, then translate into an If statement.

Example 2.4.1 A copy Shop charges \$0.05 per page up to 100 copies, and then charges \$0.03 per page.

$$C(n) = \begin{cases} 0.05n & \text{if } n \leq 100 \\ 5 + 0.03(n-100) & \text{if } n > 100 \end{cases}$$

Start: 95
increment: 1

n, # of copies	total cost
95	\$ 4.75
96	\$ 4.80
97	\$ 4.85
98	\$ 4.90
99	\$ 4.95
100	\$ 5.00
101	\$ 5.03
102	\$ 5.06
103	\$ 5.09
104	\$ 5.12
105	\$ 5.15
106	\$ 5.18
107	\$ 5.21
108	\$ 5.24
109	\$ 5.27

Example 2.4.1 A copy Shop charges \$0.05 per page up to 100 copies, and then charges \$0.03 per page up through 250 pages, and \$0.02 per page after that.

$$C(n) = \begin{cases} 0.05n & \text{if } n \leq 100 \\ 5 + 0.03(n-100) & \text{if } 100 < n \leq 250 \\ 9.50 + 0.02(n-250) & \text{if } n > 250 \end{cases}$$

Start: 245
increment: 1

n, # of copies	total cost
245	\$ 9.35
246	\$ 9.38
247	\$ 9.41
248	\$ 9.44
249	\$ 9.47
250	\$ 9.50
251	\$ 9.52

Formula bar: `=IF(B13<=100,0.05*B13,IF(B13<=250,5+0.03*(B13-100),9.5+0.02*(B13-250)))`