

In your algebra classes you have studied many types of functions, the simplest and most familiar is the linear function: $y = mx + b$.

The graph of a linear function will be a straight line, m being the slope of the line and b being the vertical intercept. An easy way to graph a linear function is to locate the b value on the y -axis, and from that point count off the slope, the slope being $\frac{\text{rise}}{\text{run}}$.

In this class, everything is a story problem. What do m and b mean when we are looking at applications? m is the "marginal" or rate of change. b is the initial value.

Examples:

A car is purchased for \$20,000 and depreciates \$1500 each year: The value of the car would be modeled by $V = 20000 - 1500t$, where t is the number of years since the car was purchased.

A sales person is paid \$800 a month plus 4% commission on sales: The sales person pay would be modeled by $P = 800 + .04s$, where s is the dollar value of the sales.

A car is 400 miles from the airport and is approaching it at 60 miles per hour: The distance to the airport would be calculated as $D = 400 - 60h$, where h = the number of hours driven.

All of these are easily translated into functions using the $y = mx + b$ formulas and then graphed using techniques from algebra. Let's try some less conventional approaches also.

Say a painting is worth \$3000 when it was 3 years old and is worth \$15,000 when it is 30 years old. We will assume its value is increasing at a constant rate. We could trot out our slope formula: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{15000 - 3000}{30 - 3} = \frac{12000}{27} = \444.44 per year, then use $y - y_1 = m(x - x_1)$ to find b . We could also sketch and get an estimate from a graph. Another approach: How much is the price increasing every 3 years? $3(444.44) = 1333.33$. So if it was worth 3000 when it was 3 years old, and it increased in value \$1333.33 in the first 3 years it must have been worth $3000 - 1333.33 = \$1666.67$ initially. Giving us a model $V(t) = 444.44t + 1666.67$.

Business Applications of Linear function: Revenue, Cost, Profit.

Revenue is the amount of money coming into a business from its sales. Other terms for revenue are income, proceeds, takings.

Costs for a business are often grouped into two types: fixed and variable. Fixed costs might include such things as rent, salaries, insurance, etc. Fixed costs do not vary with the number of items produced. Variable costs would include materials that go directly into the number of items produced. Note: There is a very general and simplified definition of costs, every business model will have different distinctions.

Profit is what is left over from our income after our expenses have been paid. Profit may be either a positive or a negative value. When profit is zero it is called the break-even point, the number of items that must be produced and sold for a business to start making money rather than losing it.

Profit = Revenue – Cost.

Let's look into a simple business model. We are going to sell tamales: Our expenses are \$100 monthly operating fee and \$.75 ingredients per tamale. We are going to charge \$1.25 for each tamale. How many do we need to make/sell to start making a profit? If we do nothing, it will cost us \$100 so we need to start moving some product.

A common sense approach: take a guess. But first, what type of thing are we guessing? Dollars? Days? Tamales? If we are taking a common sense approach we still need to know what we are looking for. Let's say we sell 100 tamales. Will we make a profit?

Costs: at \$.75 per tamale, how much will we spend on ingredients for 100 tamales? \$75, is that our only expense? No, we have that \$100 operation fee so we are looking at \$175 to get these tamales on the market. How much will we make once we have sold all of them? At \$1.25, that would be \$125. Now think carefully, have we lost \$50 or do we have a profit of \$50?

Profit = Revenue – Costs = $125 - 175 = -50$. What does negative profit imply? We need to make more money...we are not covering our expenses. Of course to sell more tamales we have to make more tamales, where do we break even?

Ignoring the operating fee for the moment, the tamales cost us \$.75 in ingredients and we sell them for \$1.25, so we have an extra \$.50 per tamale. If we need another \$50 to cover our expenses, how many more tamales is that? 100 more. So we need to sell a total of 200 tamales to break even and then we start making a profit if we sell more than 200.

We could also use algebra:

We have fixed costs of \$100 and variable costs of \$.75 per tamale. If n is the number of tamales we produce then our cost model is: $C(n) = 100 + .75n$

Each tamale is sold for \$1.25, this is the money coming into our business, our income or revenue: $R(n) = 1.25n$

From these two, we can calculate our profit formula: Profit = Revenue – Costs = $(1.25n) - (100 + .75n) = P(n) = .5n - 100$

To break-even, profit equals 0: $0 = .5n - 100 \rightarrow n = 200$

Now watch the video to use Excel to graph the same situation.

