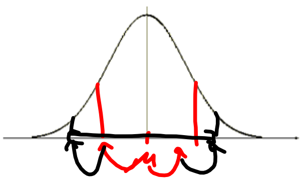
THE NORMAL DISTRIBUTION

Rules of Thumb:

$\bar{x}$ μ
s σ

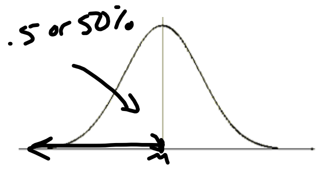
1. About 68% of the values lie within one standard deviation of the mean.
2. About 95% of the values lie within two standard deviations of the mean.
3. About 99.7% of the values lie within three standard deviations of the mean.



The probability that the random variable has a value in the interval (a, b) is given by the area under the curve between the lines $x = a$ and $x = b$.

The shape of the curve is determined by its mean and standard deviation.

The curve is symmetric about its mean (centered at the mean), the vertical line cuts the distribution in half and P(x being less than the mean) = P(x is greater than the mean) = .5

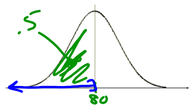


Find using the Rules of Thumb:

NORMAL CURVE PROPERTIES

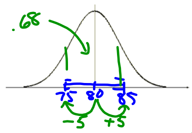
Say a test score has a normal distribution with a mean (μ) of 80 and a standard deviation of 5. σ

Probability that a test score is less than 80: $P(x < 80)$ = 50%

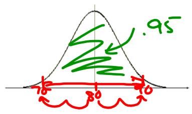


Probability that a test score is between 75 and 85: $P(75 < x < 85)$ = 68%

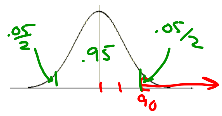
68% of the data will be within one standard deviation (5 pts) of the mean (80 pts)



Probability that a test score is between 70 and 90: $P(70 < x < 90)$ = 95%

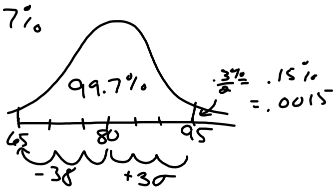


Probability that a test score is greater than 90: $P(x > 90)$ = 2.5%

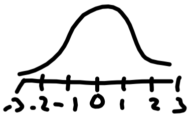


P(65<x<95) = 99.7%

80+3(5)=95
80-3(5) = 65



THE STANDARD NORMAL DISTRIBUTION



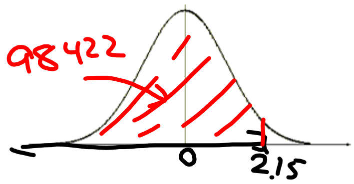
(The *Z-curve*: mean = 0 and standard deviation = 1.)

The table gives approximate values of the probabilities for a variable with a standard normal distribution to be less than z, $P(z < c)$.

Examples:

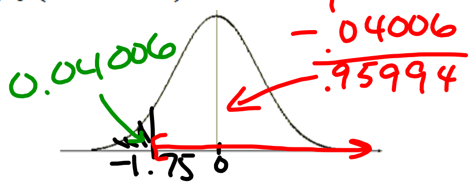
Let Z be a continuous random variable with standard normal distribution. Use the table to find:

a) $p(Z \le 2.15)$ = 98.4%

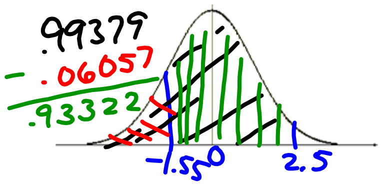


b) $p(-1.75 \le Z)$ ≈ 96%

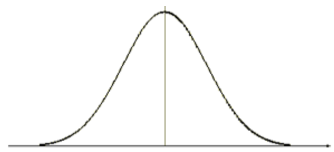
$z \ge -1.75$



c) $p(-1.55 \le Z \le 2.50)$

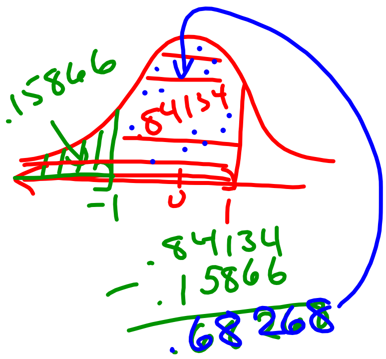


d) $p(Z \ge 1.80)$



Pop over to Excel for a minute and look at =NORMSDIST(z)…be sure to get the s in there

z	P(<z)	z	P(<z)	z	P(<z)	z	P(<z)
-4.00	0.00003	-2.00	0.02275	0.00	0.50000	2.00	0.97725
-3.95	0.00004	-1.95	0.02559	0.05	0.51994	2.05	0.97982
-3.90	0.00005	-1.90	0.02872	0.10	0.53983	2.10	0.98214
-3.85	0.00006	-1.85	0.03216	0.15	0.55962	2.15	0.98422
-3.80	0.00007	-1.80	0.03593	0.20	0.57926	2.20	0.98610
-3.75	0.00009	-1.75	0.04006	0.25	0.59871	2.25	0.98778
-3.70	0.00011	-1.70	0.04457	0.30	0.61791	2.30	0.98928
-3.65	0.00013	-1.65	0.04947	0.35	0.63683	2.35	0.99061
-3.60	0.00016	-1.60	0.05480	0.40	0.65542	2.40	0.99180
-3.55	0.00019	-1.55	0.06057	0.45	0.67364	2.45	0.99286
-3.50	0.00023	-1.50	0.06681	0.50	0.69146	2.50	0.99379
-3.45	0.00028	-1.45	0.07353	0.55	0.70884	2.55	0.99461
-3.40	0.00034	-1.40	0.08076	0.60	0.72575	2.60	0.99534
-3.35	0.00040	-1.35	0.08851	0.65	0.74215	2.65	0.99598
-3.30	0.00048	-1.30	0.09680	0.70	0.75804	2.70	0.99653
-3.25	0.00058	-1.25	0.10565	0.75	0.77337	2.75	0.99702
-3.20	0.00069	-1.20	0.11507	0.80	0.78814	2.80	0.99744
-3.15	0.00082	-1.15	0.12507	0.85	0.80234	2.85	0.99781
-3.10	0.00097	-1.10	0.13567	0.90	0.81594	2.90	0.99813
-3.05	0.00114	-1.05	0.14686	0.95	0.82894	2.95	0.99841
-3.00	0.00135	-1.00	0.15866	1.00	0.84134	3.00	0.99865
-2.95	0.00159	-0.95	0.17106	1.05	0.85314	3.05	0.99886
-2.90	0.00187	-0.90	0.18406	1.10	0.86433	3.10	0.99903
-2.85	0.00219	-0.85	0.19766	1.15	0.87493	3.15	0.99918
-2.80	0.00256	-0.80	0.21186	1.20	0.88493	3.20	0.99931
-2.75	0.00298	-0.75	0.22663	1.25	0.89435	3.25	0.99942
-2.70	0.00347	-0.70	0.24196	1.30	0.90320	3.30	0.99952
-2.65	0.00402	-0.65	0.25785	1.35	0.91149	3.35	0.99960
-2.60	0.00466	-0.60	0.27425	1.40	0.91924	3.40	0.99966
-2.55	0.00539	-0.55	0.29116	1.45	0.92647	3.45	0.99972
-2.50	0.00621	-0.50	0.30854	1.50	0.93319	3.50	0.99977
-2.45	0.00714	-0.45	0.32636	1.55	0.93943	3.55	0.99981
-2.40	0.00820	-0.40	0.34458	1.60	0.94520	3.60	0.99984
-2.35	0.00939	-0.35	0.36317	1.65	0.95053	3.65	0.99987
-2.30	0.01072	-0.30	0.38209	1.70	0.95543	3.70	0.99989
-2.25	0.01222	-0.25	0.40129	1.75	0.95994	3.75	0.99991
-2.20	0.01390	-0.20	0.42074	1.80	0.96407	3.80	0.99993
-2.15	0.01578	-0.15	0.44038	1.85	0.96784	3.85	0.99994
-2.10	0.01786	-0.10	0.46017	1.90	0.97128	3.90	0.99995
-2.05	0.02018	-0.05	0.48006	1.95	0.97441	3.95	0.99996



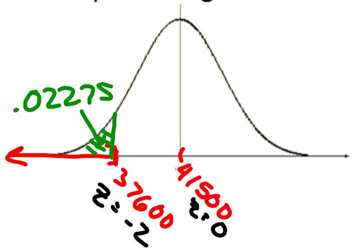
Areas under other Normal Distributions

If we have a normally distributed random variable x with mean μ and standard deviation σ, we can transform it to *standard* normal using

$$z = \frac{x-\mu}{\sigma}, \text{ in words: } z = \frac{actual\ value - mean}{standard\ deviation}$$

This is called the *z*-score; it allows us to use the standard normal table to compute probabilities.

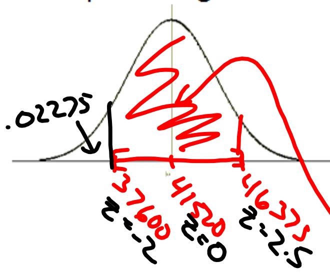
The lifetime of a certain brand of passenger tire is approximately normally distributed with a mean of 41,500 miles and a standard deviation of 1950 miles. What percentage of this brand of tires will have lifetimes less than 37,600 miles?

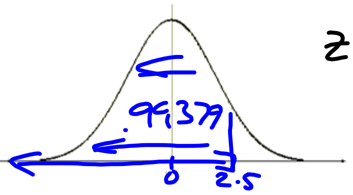


$$z = \frac{37600-41500}{1950} = -2$$

About 2.275% of the tires will fail before 37,600 miles.

What percentage of tires will have lifetimes between 37,600 and 46,375?

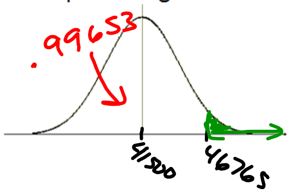




$$z = \frac{46375-41500}{1950} = 2.5$$

$$\begin{array}{r} .99379 \\ - .02275 \\ \hline .97104 \end{array}$$

What percentage of tires will have lifetimes longer than 46,765?

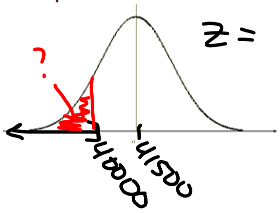


$$z = \frac{46765-41500}{1950} = 2.7$$

$$\begin{array}{r} 1 \\ -\ .99653 \\ \hline = .00347 \end{array}$$

or .35%

Supposed the company guarantees tires to last at least 40,000 miles and will replace any tire that does not last this long. What percentage of tires will have to be replaced?



$$z = \frac{40000-41500}{1950} = -.76923$$

.22089

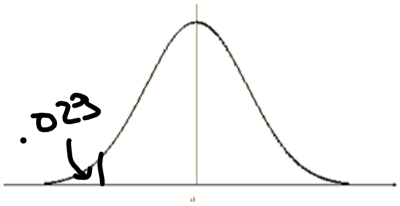
about 22.1% of the tires will need be replaced under warranty.

If the cost to replace each tire is \$86, how much will the company expect to pay on each lot of 1000 tires?

22.1% of 1000 tires is 221 tires

$$\begin{array}{r} 221 \\ \times\ \$86 \\ \hline \$19006 \end{array}$$

Sound like too much? At what number of miles should the company set as the cut off for the warranty if they want to spend no more than \$2000 to replace tires in a lot of 1000 tires?



\$2000 ÷ \$86 = 23. . . .

23 tires is what % of 1000 tires?

2.3% or .023

$z \approx -2$ or -1.99539

from chart using .02275

found using NormsInv(.023)

$$z = \frac{x-\mu}{\sigma}$$

$$1950\,(-1.995) = \left(\frac{x-41500}{1950}\right)1950$$

$$\begin{array}{rcl} -3891.016 &=& x-41500 \\ +41500 & & +41500 \\ \hline 37608 &=& x \end{array}$$

We should warranty the tires to last 37,600 miles

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normalcdf(⁻2,10)
       .977249938
normalcdf(⁻10,⁻2
)
       .022750062
█
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