MTH245 Unit 4 Module 2 Expected Value

Expected value is important to consider when you have a situation with different possible outcomes occurring and you have some idea how likely they are to occur. Such as if you have a problem gambling habit. Expected value is for looking at what happens in the long run. If you go to a casino occasionally, that's entertainment, if you are there every weekend you really need to look at the expected value of the game. Expected value is how much you expect to gain, or lose, on average over time.

Insurance rates are determined by looking at the driving history of a large number of people. Each demographic group is assigned a rate based on the likelihood of someone in the group having an accident. You may be an excellent driver but if you are a 20-something male you are likely paying a lot for car insurance because of other driver's poor records.

The Oregon lottery was originally established as a way to raise money for economic development in Oregon. (Now it funds a lot of different things such as schools and parks.) Games were designed to pay 50 cents on the dollar. So if you bought 1000 scratch-its for $1000, you would expect to win back around $500 of what you spent. Any individual ticket could be a jackpot, but over the long run you would only be winning back half of what you spend on average.

Quotes from FAQ on the OR lottery website:

13 - How much is the Lottery required to return to players in the form of prizes?

The law requires that at least 50% of the Lottery's total annual sales be returned to the public in the form of prizes. The Lottery currently pays out Traditional game (Megabucks, Powerball, Scratch-its, Sports Action, Pick 4, Keno, Breakopens) prizes at a combined rate of 64%, which means that overall, of 64 cents of every dollar played on Traditional games goes back to players in prizes. Video Lottery game prizes pay out at a combined rate of 93%.

3 - The Scratch-it game I played had odds of 1 in 4. Doesn't that mean I should get a winning ticket every fourth ticket purchased?

The odds stated on the back of all Scratch-it tickets are the overall odds of winning a prize in that game. If the overall odds of winning a prize in a game are 1 in 4, it means that if 4 million tickets are printed, 1 million will be winners. Winning tickets are then randomly placed throughout the Scratch-it games. As with all of our games, receiving one of those winning tickets is simply a matter of chance. While it's possible to get a string of non-winners in a row, it's also possible to buy a similar number of tickets and get several winners in a row. While we can't guarantee that everyone will be a winner, we can guarantee that each player has a fair and equal chance of winning a prize.

Also from the Oregon Lottery website:

- Raffle tickets went on sale January 22. Purchase your $10 ticket at any Lottery retailer or Lottery To Go™. Tickets are limited (only 250,000), so get yours early.
- One $1 million prizes
- Ten $20,000 prizes
- One thousand $100 prizes

There is one prize worth $1,000,000 out of 250,000 tickets so the P(winning one million) = 1/250,000.

The probability for the $20,000 and $100 prizes can be calculated the same way.

To find the probability of winning nothing, note that there are 250,000 tickets sold and that 1 + 10 + 1000 = 1011 tickets will win a prize. That leaves 250,000 – 1011 = 248,989 tickets to be "losers".

A probability distribution table for x="amount I win" would be:

x	\$1,000,000	\$20,000	\$100	0
P(x)	$\frac{1}{250,000}$	$\frac{10}{250,000}$	$\frac{1000}{250,000}$	$\frac{248,989}{250,000}$

or, if you prefer decimals:

x	\$1,000,000	\$20,000	\$100	0
P(x)	0.000004	0.00004	.004	.995956

Notice that .000 004 + .000 04 + .004 + .995 956 = 1, which should always be the case for any probability distribution. Something has to happen so the sum of the likelihood of all possibilities should be 1 (100%).

Expected value is found by multiplying the "value" of an outcome by the probability of that outcome and then adding up all of these products. We have 4 possible outcomes, and the probability of each of those 4 outcomes.

$$EV = \$1{,}000{,}000 \cdot .000\,004 + \$20{,}000 \cdot .000\,04 + \$100 \cdot .004 + 0 \cdot .995956 = \$5.20$$

So if we were to play this game a "large" number of times, we would expect to win, on average, \$5.20 per ticket. How much did each ticket cost? \$10. So, on average, we end up *losing* \$4.80 per ticket. (But of course, you may have a \$1,000,000 ticket.)

Then the expected value of the **prize** is \$5.20, but the expected value of the winnings, or the expected value of the **profit**, is **-**\$4.80. With a negative expected profit, the game is in the favor of the state, rather than the player.

By the way, it is impossible to win a prize of \$5.20 in this particular game, \$5.20 is an average. Do not think of expected value as an outcome for a single ticket, or "trial", or "experiment". It is the theoretical long term average if we play a large number of times, but it is **not** the typical outcome. The typical outcome would be winning \$0, because that is what most of the tickets will be worth.

Another way to think about this would be to think of the state revenue. They sell 250,000 tickets for \$10, that's \$2,500,000 and they give back a total of \$1,300,000 in prizes. That means that the state collected \$1,200,000 from this particular game to fund various agencies without taxes.

x	P(x)
-$1	.25
$0	.50
$3	.25

Let's consider a possible gambling game: Pay $1 to play, toss a coin twice. If no heads, the house pays nothing (you have a loss of $1); if one head appears the house pays $1 (no gain, no loss to you); two heads will pay $4 (gain of $3). The probability distribution is a list of all values in the sample space: -1, 0, 3 and the probability of each of those possible outcomes.
(Need to review? See Unit 3 Module 1: P(getting two heads) = (.5)(.5) = .25, P(one head)= P(HT)+P(TH)=(.5)(.5)+(.5)(.5)=.5, etc.) Note that the sum of the P(x) is 1.

Once we have the probability distribution it is easy to calculate the expected value: multiply each outcome by the likelihood of it occurring and sum up the results.

EV = -1(.25) + 0(.5) + 3(.25) = .5
In the long run, we would expect to win $.50 per game, **positive** $.50. In this case, the game is in favor of the player. It would pay to play, but no casino would offer a game that is in favor of the player.

Note how in the lottery game I adjusted for the ticket price at the end while for the dice game I took care of the "pay to play" in the table. Either method is fine--sometimes the numbers are "neater" one way or the other. But do keep track of the difference between the **prize** and the **profit** (or loss) if there is a fee for the game.

One last example: A farmer is deciding whether to plant wheat or rye grass this year. The value of the crop (quality and quantity) will depend on the weather, whether the summer is dry, damp or mixed. The probability of the weather being dry, damp or mixed, and the value of the crop is given below:

Weather	Probability of weather	Value of wheat in such conditions	Value of rye grass in such conditions
Dry	.6	$300	$900
Damp	.1	$1000	$200
Mixed	.3	$600	$500

Expected value if plant wheat: 300(.6) + 1000(.1) + 600(.3) = $460
Expected value if plant rye grass: 900(.6) + 200(.1) + 500(.3) = $710
In the long run, which crop is a better choice for the farmer? The rye grass as it has a higher expected value.

Let's go to the video and look at how we can use Excel to help with the calculations. The function =SUMPRODUCT(array1, array2,...) will be useful.