**MTH245 Unit 1 Module 1 Algebra Basics and Percents**

In algebra classes we often talk about there being 3 ways to approach a problem: symbolically, graphically and numerically. And we rarely, if ever, use the numeric (quantitative) approach. This class is about solving problems that do not lend themselves to algebraic solutions. There still will be some algebra used, but what we want to develop in this class is the ability to problem solve—how to analyze a problem and come up with a logical approach to find a solution. The problems are all applied, and each is unique. You will need to use persistence and creativity. This is not a class where you see an example and then do 10 variations of the same problem for homework.

We will use Microsoft Excel in this class. We will start with the basics and will not assume that you are already an expert.

## Some Basics of Algebra

This will be a very quick recap of some of the algebra techniques we will use in this course. It should all be familiar but if you find some unfamiliar topics you may want to pick up an algebra textbook and review what you are "rusty" on.

### Scientific Notation

Scientific notation is useful when working with very large or very small numbers.

Example 1:

| | Standard Notation | Scientific Notation (Traditional Format) | Scientific Notation (Excel Format) |
|---|---|---|---|
| a) | 26,000,000 | $2.6 \times 10^7$ | 2.6E7 |
| b) | 1,300,000,000 | $1.3 \times 10^9$ | 1.3E9 |
| c) | 0.0000056 | $5.6 \times 10^{-6}$ | 5.6E-6 |

### Fractions

To add, or subtract, fractions the fractions need to have a common denominator. If they do not, both the numerator and denominator of a fraction may be multiplied by a common factor to rewrite it in an equivalent form that has the desired denominator. Multiplying fractions is easier; you just need to multiply the numerators and denominators, no common denominator needed! To divide a fraction by a second fraction, invert the second fraction and them multiply the result.

Example 2:

| | | |
|---|---|---|
| a) $\frac{1}{2}+\frac{3}{4}=\frac{1}{2}\cdot\frac{2}{2}+\frac{3}{4}$ $=\frac{2}{4}+\frac{3}{4}=\frac{5}{4}$ | b) $\frac{1}{2}\cdot\frac{3}{4}=\frac{3}{8}$ | c) $\frac{1}{2}\div\frac{3}{4}=\frac{1}{2}\cdot\frac{4}{3}$ $=\frac{4}{6}=\frac{2}{3}$ |

**Properties of Exponents**

| | | | |
|---|---|---|---|
| $a^m \cdot a^n = a^{m+n}$ | $\frac{a^m}{a^n} = a^{m-n}$ | $a^0 = 1$ | $a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$ |
| $(a^m)^n = a^{m\cdot n}$ | $(ab)^m = a^m b^m$ | $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ | |

Example 3: $\frac{y^{-10}y^3}{y^{-4}} = \frac{y^{-10+3}}{y^{-4}} = \frac{y^{-7}}{y^{-4}} = y^{-7--4} = y^{-7+4} = y^{-3} = \frac{1}{y^3}$

Example 4: $(a^{-4}b^6c^{-2})^3 = a^{-4\cdot3}b^{6\cdot3}c^{-2\cdot3} = a^{-12}b^{18}c^{-6} = \frac{b^{18}}{a^{12}c^6}$

Example 5: Several years ago, the population of the United States was approximately 300 million. At that time the combined retail, accommodation and food service sales was $4.5 trillion. What was the rate of spending per capita?

$\frac{sales}{person} = \frac{4.5\ trillion}{300\ million} = \frac{4.5\times10^{12}}{300\times10^6} = \frac{4.5\times10^{12}}{3\times10^8} = \frac{4.5}{3}\cdot\frac{10^{12}}{10^8} = 1.5\times10^4$=$15,000 per person

**Percents**

The word “percent” means “divide by 100”. So 5% is 5/100 which reduces to 1/20 or 0.05.

When working with percents “of” means “multiply”

20% of $80 is $20\% \cdot 80 = 0.2 \cdot 80 = 16$ or $16

A pair of shoes is usually $80 but is marked down 20%, then you will save $16. (80 – 16 = $64 for sale price)

The wholesale cost of a book is $84 but is marked up 25% for the retail price. What is the retail price?

25% of 84 = ¼ * 84 = 21, mark up is $21 so the retail price is $84 + $21 = $105

Keep in mind that sometimes fractions are more convenient than decimals to work with. Personal favorites:

1% = 1/100      5% = 1/20      10% = 1/10      50% = ½      100% = 1      200% = 2

35 is 5% of what number?  35 = 1/20 * p, so p = 20 * 35 = 700
Multiplying by 20 is easier to do in one's head than dividing by 0.05.

When working with percents, it is important to keep track of what it is a percentage of.
A store purchases a book for $100 and sells it for $120.  How much is the markup?  $20.
The mark up is what percent of the **original** price?  20 = what % of 100. $20 = x \cdot 100$, solve for x and convert to a percentage: 20%.
Alternately we could look at the mark up as being a percentage of the **retail** price: 20 = what % of 120? $20 = x \cdot 120$, solving for x and converting to a percentage would be 16.7%

Carefully compare these two problems:
Water in a tank **de**creases from 200 gallons to 100 gallons.  What is the percent decrease?
    100 is what % of 200? (50%)
Water in a tank **in**creases from 100 gallons to 200 gallons.  What is the percent increase?
    100 is what % of 100? (100%)

Another one:  If gas costs $3 a gallon and increases by 100%, how much will it now cost? ($6)

During tough economic times your company asks you to take a wage cut of 20% for 6 months and then it will give you a 20% raise.  Will you be back to your original wage?
It does not matter what your original wage is, so we will pick a convenient value for the salary of $50,000: 20% of 50,000 = .2 * 50,000 = 10,000 you have a $10,000 wage cut, so your temporary salary is $40,000.
Six months later you get your raise: 20% 0f 40,000 = .2 * 40,000 = $8000 your raise is only $8000 so your new wage is $48,000.  Your new salary is a permanent decrease of $2000.

A little bit of algebra can make this process more streamlined.  This is important as we will be doing this a lot.  Returning to the example of the shoes that are on sale for 20% off, if the shoes were discounted 20% that would mean we must be paying the remaining 80%, right?  80% of $80 = .8 * 80 = $64 for the sale price.  Same answer, but faster.

A handy pair of formulas when working with percentage changes is:

| | |
|---|---|
| % change increase | new value = old value (1 + % change) |
| % change decrease | new value = old value (1 – % change) |

Look at the salary problem again: original salary was $50,000 then we had an increase of 20% followed by a decrease of 20%
50,000(1 - .2)(1 + .2) = 50,000(.8)(1.2) = 50,000(.96) we ended up at 96% of our original salary

Another one: Store marks up wholesale prices by 25%, in addition consumers are charged 5% sales tax. How much above retail is the consumer paying? Let's say an item costs $100 wholesale:
100(1+.25)(1+.05)= 100(1.25)(1.05) = 100(1.3125)The consumer pays 31.25% more than wholesale.

A variation: Joe gets a 10% raise every year, his starting wage is $8 an hour, what is his wage after 3 years? 8(1 + 0.1)(1+0.1)(1+0.1) = $8(1.1)^3$ = $10.648.

Notice that he gets the same percentage raise each year, but **not** the same dollar amount raise. The first year his raise is 10% of 8, which is 80 cents. The second year his 10% raise is based on his higher salary of $8.80, so his second raise is 88 cents. Each year he gets a larger raise in terms of the dollar amount.

You may recognize the formula we are using here from your earlier algebra classes. This is the same as $A = P(1+r)^t$

Jim started at a wage of $9 an hour and 2 years later is earning $11, If his wage rose by the same percent each year, what was the percentage rate? the unknown value is the percentage increase, let's call that p. Then, for his $9 wage to grow to $11 in two years we would use:

$9(1 + p)^2 = 11$, solving we would divide both sides by 9, square root both sides and subtract 1 to find p = 10.55%